# \#3, Due Monday 08/22 at 6pm 

## Last updated on $08 / 15 / 2016$

## Part 1

Section 4.3: $10,11,12$

Section 5.1: $\quad 1(\mathrm{a})(\mathrm{b})(\mathrm{d})(\mathrm{e})(\mathrm{f})(\mathrm{i})(\mathrm{j})(\mathrm{k}), 3(\mathrm{a})(\mathrm{b})(\mathrm{c}), 4(\mathrm{f}), 8(\mathrm{a})(\mathrm{b}), 17(\mathrm{a})(\mathrm{b})(\mathrm{c})$

Section 5.2: 1(a)-(g)(All of them!), 2(a),3(a),20

Section 5.4: $1(\mathrm{a})(\mathrm{f})($ Hint: See Section 4.3, Exercise 24), 2(a)(c)(e), 18(a)(b)(c)

Section 7.1: $3(\mathrm{a})(\mathrm{c})$ (You only need to find the Jordan canonical form of $T$ in (a) and (c))

Section 7.2: $1(\mathrm{a})(\mathrm{d})(\mathrm{g}), 3(\mathrm{a})-(\mathrm{e})($ All of them! $), 3(\mathrm{a})(\mathrm{c})(\mathrm{d})(\mathrm{e})$

## Part 2

1. Prove that for any $A, B \in M_{n \times n}(F), A B$ and $B A$ have the same eigenvalues. More precisely, for any $\lambda \in F, \lambda$ is an eigenvalue of $A B$ if and only if $\lambda$ is an eigenvalue of $B A$.
2. $T: V \rightarrow V$ is a linear transformation, $x \in V$ such that $T^{k-1}(x) \neq 0, T^{k}(x)=0$. Prove that $\beta=\left\{x, T x, T^{2}(x), \cdots, T^{k-1}(x)\right\}$ is linearly independent. Let $W=\operatorname{Span} \beta$, then $W$ is a $T$-invariant subspace, what is $\left[\left.T\right|_{W}\right]_{\beta}$ ?
3. Find an explicit formula for $a_{n}$ where $a_{n}$ is defined by

$$
a_{1}=a_{2}=1
$$

and

$$
a_{n}=a_{n-1}+a_{n-2}
$$

for $n \geq 3$.

